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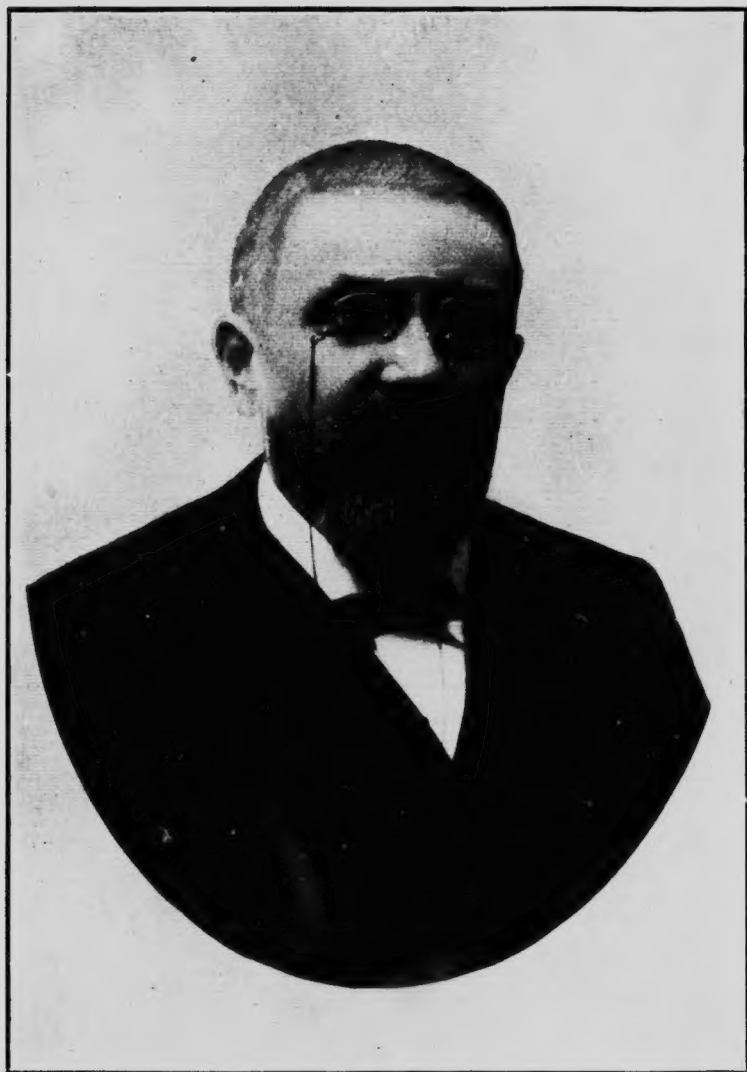
JULES HENRI POINCARÉ

BY

ALFRED T. DELURY, M.A.,
PROFESSOR OF MATHEMATICS, UNIVERSITY OF TORONTO

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PLATE XXX.



JULES HENRI POINCARÉ
Born April 29, 1854. Died July 17, 1912.

Journal of the Royal Astronomical Society of Canada, 1912

JULES HENRI POINCARÉ*

BY ALFRED T. DELURY

WHEN in July last it was announced that Poincaré was no more, workers in many fields of enquiry were, through the tragic circumstance of death, startled into a realization of the debt their science owed to him and of the loss suffered in the passing of so vast a soul. For many years the science in which our Society is chiefly interested had claimed no small part of his attention, and his contributions to it, while for the most part outside the range of the general student, are numerous and profound. Fitting is it, then, on resuming our meetings for the year, that we devote an evening to a review, necessarily inadequate, of his work, and to the contemplation of a rich, full and stimulating, if all too short, life.

Henri Poincaré was born April 29, 1854, at Nancy, of Lorraine, a part of France notably fecund of genius. His family were, on the one side, small land-holders warmly attached to

* Report of an address delivered before the Royal Astronomical Society of Canada, November 5, 1912.

country interests, on the other side, of the professional class, lawyers, physicians, pharmacists and soldiers. His father was a practising physician and a member of the Faculty at Nancy. With the family lived an uncle, a graduate of the *Ecole Polytechnique*, an officer of the state in the department of *Ponts et Chaussées*. The youthful Poincaré was heir then to the double tradition of learning and provincial simplicity of life.

As a child he was alert, even precocious, and stories illustrative of his early awakened interest in science survive. One only will be told, and this finds him at an early age. The story runs that when only nine months old, one evening, as darkness fell, his eye caught a star and he called his mother's attention to it; as others shone out he espied them and became excitedly intent in his search for others, so much so that he was with difficulty quieted, and brought into a mood for sleep. In this, one may choose to see a presage. At the age of five a serious illness threatened for a time to deprive him of the power of speech; it left him weak, and intensified in him a certain native shyness. Illness often has a maturing influence, and it seems to have affected the child Poincaré in this way. He was less drawn to childish sports and more to reading, and the reading of books dealing with simple science rather than fairy tales and romances. His first formal instruction was given at his home in a charmingly informal manner by an old retired and scholarly schoolmaster. Everything was treated as a matter to be talked about, and the talk was of everything. The result was that when he had to take his place in school, and feel the yoke of method, he brought to his studies a certain general knowledge and a free and quickened curiosity.

He entered the *lycée* of his native city at the age of nine and followed his studies there for eleven years. He showed an aptness in all his courses, with a marked leaning to letters. When, however, he reached the class in which the pupils became acquainted with geometry, there was no longer any doubt as to what studies were to claim him. His teacher hastened to tell his mother that her son was destined to be a great mathematician,

and each year of work at the lycée strengthened the faith in the prediction. It was while he was still a lycéean that he was a witness of the humiliation of his native land, Nancy itself being occupied by the victorious Germans. With some of the hideous realities of war he came into close contact while assisting his father who, as physician, was caring for the sick and wounded. Scanning the war bulletins and interpreting to himself the reports of operations in the only papers that were within his reach, he came to understand the German language. Nancy was in the territory occupied by the Germans after the war, and the last of his many collegiate honors were conferred as, in the same hall, was announced the withdrawal of the foreign troops. He had taken, in 1871, the degrees of Bachelor of Letters and Bachelor of Sciences, and in 1873 in the competitive examinations for entrance into the *École Polytechnique* and the *École Normale* he had been ranked first and fifth respectively. His college days at Nancy were at a close; the time for the decision as to his life's work had come.

He chose to enter the *École Polytechnique*, the great school leading to careers in the army and in such services as are administered by the department of *Travaux Publics*, the *École Normale* opening up immediately careers in the higher teaching and favoring, perhaps, studies pursued for scientific rather than practical ends. What decided Poincaré, without marked aptitude for the practical, in electing as he did, it is difficult to say. His course for the two years in the *École Polytechnique* was a brilliant one, in all that related to purely scientific studies, and of these mathematical studies made up the greater part. Originality and resource characterized his work, and his very eccentricity of manner, and method of study seemed to mark him as one destined to enlarge the boundaries of science.

When in 1875 he had completed the course, he decided to enter the *Service des Mines*, and became enrolled as *Élève Ingénieur* at the *École des Mines*. This meant a great change in studies and in mode of life. His course allowed or required him to make visits, for observation and study, to other countries.

In the freer life which he now enjoyed he became responsive to new and varied influences and the stimulus that they bring. His interest in pure science did not abate, and the year 1876 saw him *Licencié ès Sciences*; in 1879 he was awarded the degree of *Docteur ès Sciences*, his thesis being *Sur les propriétés des fonctions définies par les équations aux différentielles partielles*. In 1879 he was named *Ingenieur Ordinaire des Mines*, and was for some time in charge of the mines at Vassoul. In the fall of 1879 he was, by request, transferred to the state service in the higher teaching and placed in charge of the Course in Analysis at the Faculty of Sciences at Caen.

These were important years for Poincaré, the years in which the peculiar character of his genius was to reveal itself. In France the activity in mathematical work was very striking. Time was still adding to the suggestive power of Hermite, and near to him were many auxiliary sources, stars shining amid the effulgence,—and a star of the first order was rising. In Germany, the inspiration of the discoveries and teaching of Weierstrass was expressing itself in results that were quickening mathematical interest in every land. These were the favoring conditions,—favoring in that they served to accentuate his individuality and choice, and not to assimilate his ideas and make them tributary to a school of thought.

His early papers—his thesis and a memoir offered in competition for the *Grand Prix des Sciences Mathématiques*—were on differential equations, that meeting ground for so many mathematical theories. This memoir, though it did not call for the award, elicited from Hermite, who reported on the papers submitted, the warmest commendation, with the counsel that the writer follow up certain lines of enquiry then traced. The two papers in question show that Poincaré had come to realize that advances in the treatment of differential equations were dependent upon advances in the general theory of functions. A reference to his work, by Hermite, in a paper contributed to *Crelle's Journal*, in December, 1880, would indicate further that he was giving thought to this theory.

In February, 1881, appeared the first of a long line of notes and papers on the theory of functions,—papers enlarging the prospect in analysis and revealing discoveries generally regarded as the richest offerings of his genius. The earliest of these notes appeared in the *Comptes Rendus*, announcing, under such titles as *Sur les fonctions fuchsienues*, *Sur les groupes Kleinéens*, *Sur les fonctions à espaces lacunaires*, the results of his investigations. The new theories were elaborated in a brilliant series of memoirs published in the earlier volumes of *Acta Mathematica*. It would be difficult here to explain, in any detail, the significance and importance of these discoveries, and I shall limit myself to the citing of only one of the most striking consequences, the fact that through the newly introduced functions the study of the most general non-uniform analytic functions of one variable is reduced to the study of uniform functions.

While these theories were being developed, Poincaré's interest in the subject that first claimed, or claimed a first place in, his attention, namely, differential equations, only increased. To these years we owe the remarkable series of papers, *Sur les courbes définies par les équations différentielles*.—(Four parts, *Liouville's Journal*, 1881-1886).

In the fall of 1881, Poincaré was named to the Faculty of Sciences at Paris as *Maitre de Conférences d'Analyse* and in 1883 the duties of *Répétiteur d'Analyse à l'École Polytechnique* were added. He became *Chargé du Cours de Mécanique Physique et Expérimentale* in the Faculty of Sciences, and the next year was appointed to the Professorship of Mathematical Physics. These later appointments reflected rather than determined the line of progress, or direction of interest, in Poincaré's mind, and, in the choice, he was faithful to the traditions of French Science, for, though striking instances to the contrary will occur to every one, we may say that in the main the French mathematicians have been able physicists, and French physicists able mathematicians.

As professor of Mathematical Physics, Poincaré, year after year, chose a new subject for his course, and brought his

strength to bear on the outstanding questions in the different branches of physical science. His lectures were published, and the charm and skill of his manner of exposition allowed to impress those who could not hear him. The favorable reviews of these volumes as they appeared show the esteem in which they were held by those competent to judge. Two of them I have at least to name, for they appear to me ideal treatments of their subjects. I refer to the *Théorie Analytique de la Propagation de la Chaleur*, and the *Théorie du Potentiel Newtonien*.

We may now turn to another field of application of analysis and in particular of differential equations, a field which might be included in mathematical physics, but which is well designated by a special title,—the field of mathematical astronomy or we may say *mécanique céleste*, including *mécanique analytique* as applied to astronomical problems. Early in 1885, appeared certain important notes in the *Comptes Rendus* indicating valuable results, obtained by Poincaré, in relation to the behavior of a fluid mass under rotation, and later in the year there was published in *Acta Mathematica* his great memoir *Sur l'équilibre d'une masse fluide en rotation*. Jacobi had shown that the ellipsoid of revolution, the received stable form of a fluid mass rotating under the action of gravity, would, under contraction and increase of angular velocity, reach a stage that marked a limit to stability when the ellipsoidal body would pass into a new series of forms, ellipsoidal but with three unequal axes, into what, in picturesque language, is called a series of cigar-shaped forms. Poincaré shows that a limit to stability is again reached when a series of pear-shaped forms would follow. To convey an idea of the significance of this memoir in an important side of astronomy let me quote from the address of Professor Darwin, when presenting to Poincaré, in 1900, the medal of the Royal Astronomical Society of London.

"This apparently abstract result elucidates the evolution of planetary systems in a very interesting way. Let us consider a rotating liquid mass slowly cooling. If the cooling is slow enough internal friction will cause the whole to revolve through-

out with the same angular velocity. At first, when the density is small, the figure will be an ellipsoid of revolution but slightly flattened; and as it cools the flattening will increase until at a certain stage the figure of revolution ceases to be a figure of equilibrium, and the ellipsoid commences to have an equatorial protuberance; it passes, in fact, into one of Jacobi's ellipsoids. The ellipsoid then lengthens until at a certain stage it begins to acquire an unsymmetrical furrow in a plane parallel to the axis of revolution, and it becomes pear-shaped, with the axis of revolution at right angles to the core of the pear. 'The larger part of the matter tends to approach the spherical form', whilst the smaller part projects from the ellipsoid at one of the extremities of the longer axis, as though it were trying to detach itself from the larger part of the mass. It is difficult to state with certainty what will happen then if the cooling continues, but one may suppose that the mass will go on deepening its furrow more and more, and then it will at last divide itself into two separate bodies by the throttling of the middle part.' It is clear that a process of this kind may have played its part in the evolution of celestial systems, and the speculation seems to meet with confirmation from the forms observed in many nebulae."

"... Poincaré's paper is one that will always mark an important epoch in the history not only of evolutionary astronomy, but of the wider fields of general mechanics."

Professor Darwin has made signal use of this theory, for based on it is his brilliant theory of the evolution of the earth-moon system.

On the problem of the three bodies, Poincaré published many important papers. His great memoir, *Sur le Problème des trois Corps, et les équations de la Dynamique*, won for him the prize founded by the King of Sweden and Norway. It was published in *Acta Mathematica* in 1890. Soon afterwards, in 1892, appeared the first volume of his important work *Les Méthodes Nouvelles de la Mécanique Céleste*. The two later volumes appeared in 1894 and 1899. As the name suggests, their subject matter is not the classical theory, but a presentation of the newer methods of

treatment of the more important problems. The theories and treatments associated with the names of Hill, Gylden, Linstedt, are discussed and examined. Where an investigator has had the *hardiesse* to make use of a process without as yet having complete logical warrant, as in the case of Hill's employing the infinite determinant, Poincaré sets himself the problem of making the foundations sure. His own contributions are developed incidentally to these critical examinations of theories.

It will serve to bring us nearer to what Poincaré had in mind in writing the *Nouvelles Méthodes*, of which Darwin has said that "it is probable that for half a century to come it will be the mine from which humbler investigators will excavate their materials." To quote from his Introduction :

"The final object of Celestial Mechanics is to determine whether the law of Newton, of itself alone, explains all astronomical phenomena. The one way to accomplish this is to make observations as precise as possible and then to compare them with the results of computation. These results are only approximate, and it would serve no purpose to compute to a higher place of decimals than the observations can reveal. It is useless then to demand of the computation a higher degree of precision than of the observations ; but one ought not to be satisfied with a lower degree. Further the approximations with which to-day we are content, will be in time insufficient.

"The time when we shall be obliged to give up the older methods is, beyond doubt, still far away ; but the theoretical worker is obliged to anticipate it, since his work must precede, and often by many years, that of the computer.

"The discussion of the convergence of these developments must claim the attention of mathematicians, first for the reasons explained, and further because the object of Celestial Mechanics is not attained when we have calculated ephemerides, more or less approximate, without being able to estimate the degree of approximation. If we arrive at a divergence between these ephemerides and the observations, it is necessary that we be able to discover whether Newton's law is in defect, or that the explan-

ation lies in an imperfect theory. It is important then to determine a superior limit to the error, and this problem has not hitherto sufficiently engaged attention. Now the methods which allow us to discuss the convergence give us at the same time this upper limit, and their importance and utility is enhanced. The reader will not then be astonished at the place which I have given them, in this work, although probably I have not derived from them all the results that might have been of service.

"I have occupied myself with these questions, and to them I have devoted a memoir published in Volume XIII. of *Acta Mathematica*. There above all I set myself the problem of presenting the few results relative to the Problem of Three Bodies which can be established in the complete rigour demanded by mathematics. This rigour it is that constitutes the value in my theorems on periodic, asymptotic, and doubly asymptotic solutions. In them may be found a firm foundation on which one may build with confidence.

"It seemed to me on the other hand, that my results would allow me to unite in a sort of synthesis the greater part of the new methods recently proposed, and this it was that decided me to undertake the present work."

In 1896, on the death of Tisserand, Poincaré chose to be appointed to the Professorship in Mathematical Astronomy and Celestial Mechanics. His lectures in the earlier years of this chair were published under the title *Leçons de Mécanique Céleste*.

One of his latest publications was his lectures on theories of cosmogony. In this we have a singularly attractive discussion of the different hypotheses that have been made to afford a foundation for an admitted evolution.

Reference should be made also to certain important papers on tides, but enough has been said to indicate that in astronomy, the aristocrat of the sciences, he has made advances that alone would render his name illustrious for all time.

On his mathematical work much more might be said, but further comments must be brief. He could not have been associated with Hermite without recognizing the contact of his investiga-

tions with the theory of forms and the higher arithmetic and algebra. His attention too was early drawn to the non-euclidian geometry. The postulate of Euclid on parallel lines had, for reasons now familiar to all, seemed to challenge proof. Implied in it is the unboundedness, and the infinitude of space, and for a long time it was not seen that this involved in it the impossibility of proof and marked it as a veritable postulate. Early in the century which saw the greater part of Poincaré's activity, the epoch-making discoveries of Lobatchevsky and Bolyai had settled the question. Poincaré realized to the full the philosophical significance of the new geometries, and, in a note recording the process of his mind in reaching certain results, bears testimony to its suggestiveness in the domain of pure analysis. He wrote a number of brilliant papers on this subject, and supplemented Hilbert's work on the foundations of geometry. For his studies in this domain he was in 1904 awarded the Lobatchevsky gold medal of the Mathematical Society of Kasan, and in 1905 the Bolyai prize by the Hungarian Academy of Sciences. To the *Analysis Situs* also he made valuable contributions.

It is now in place to say that the philosophical side of the subjects of his thought, the question of foundations, the rôle of logic, and of experience, strongly attracted him. Papers dealing with this side of the sciences were contributed to several reviews and periodicals. Certain of these were collected and published in three volumes *La Science et l'Hypothèse*, *La Valeur de la Science*, *Science et Méthode*. The literary excellence of these essays, and the boldness and directness of the speculation, won for them at once an astonishing popularity. They were translated into many languages and through them many who would otherwise have been untouched directly by Poincaré's thought, came to know something of the workings of his mind.

Of the subjects treated only one will be selected for comment, that on the nature of mathematical reasoning. The author considers the fact that the striking quality of mathematical reasoning is its logical character and asks wherein resides the fruitful principle certainly present in mathematics, while in logic

we have, to be sure, a criterion, a touchstone for argument, but no source of new life. He finds this in the method of complete or mathematical induction. This may be illustrated very simply. For example, we notice that if we have the succession of terms $1 + 3 + 5 + 7 + \dots$, the sums of 1, 2, 3, 4, $\dots$ terms are $1^2, 2^2, 3^2, 4^2, \dots$, this as far as we choose to add. We may feel that we have a warrant for assuming that the correspondence will continue but it does not follow that it *must* continue. Let us assume that the sum of n terms is n^2 , i. e., that

$$1 + 3 + 5 + \dots + (2n - 1) = n^2$$

Then the sum of $n + 1$ terms is this sum increased by the next term, or $2n + 1$. This yields $n^2 + 2n + 1$, which is $(n + 1)^2$. Thus if the correspondence holds out to the n th term it must hold for $n + 1$ terms. But it is valid for 1, 2, 3, 4, $\dots$ terms: hence it must hold for 5, 6, 7, 8, $\dots$ or any number of terms. This induction is accordingly perfect. It comprehends an infinity of syllogisms and in it lies the element necessary to creation in the domain of mathematics. Poincaré gives a very illuminating treatment of the elementary laws of arithmetic, and his whole argument is very stimulating.

It would go without saying that the work of this keen investigator would bring him many honours, many acknowledgments of distinction. In 1887 he became a member of the Academy of Sciences and in 1908 the *Académie Française* opened its doors to him. He was a member of all the principal scientific societies and academies of the two continents, and was awarded many prizes and medals by scientific bodies. He presided over the meetings of the Third International Congress of Mathematicians in Paris, 1900.

While at Rome, at the Mathematical Congress of 1908, he was taken seriously ill, and was unable to be present at the later sessions of the meeting. He was, in time, able to resume his work, and his many friends came to feel that, comparatively young, he had many years before him. This, however, was not to be. The ailment returning or persisting, a surgical operation

was counselled. For a short time after the operation, all seemed well, but unexpectedly and suddenly the end came.

Mathematicians, in respect to the outstanding character of their genius have been, so far as they have been creative, assigned to two classes, the intuitionists and analysts. One of the most interesting of Poincaré's philosophical papers deals with this very matter, and in considering the veritable nature of intuition, he would claim that intuition is essential to invention. He distinguishes the intuition of pure number, necessary to the creative analyst, from sensible intuition, or we may say imagination. The distinction exists but is not easily marked; one may name striking cases illustrative of each class, and cases in which each class might be illustrated. The variety presented by Poincaré's own achievements adds to the difficulty of realizing that the two kinds of intuition are sharply separated. We may then not err if we regard his intellectual strength and activity as having been sustained and stimulated by both sources.

The loss to the scientific world, in the death of Poincaré, is keenly felt though it cannot be estimated. The touching and eloquent tributes paid his memory, at his grave, testify to the great variety of his intellectual interest, and to the outstanding quality of his work in different domains, as well as to the charm and beauty of his character. From a later and incidental appreciation you must allow me to quote. At the opening meeting of the International Congress of Mathematicians, in August last, Sir George Howard Darwin extended on behalf of the Cambridge Philosophical Society a welcome to the ancient city of colleges. In the course of his address he said:—

"The Science of mathematics is now so wide and is already so much specialized that it may be doubted whether there exists to-day any man fully competent to understand mathematical research in all its diverse branches. I, at least, feel how profoundly ill-equipped I am to represent our Society as regards all that vast field of knowledge which we classify as pure mathematics. I must tell you frankly that when I gaze on some of

the papers written by men in this room I feel myself in the same position as if they were written in Sanskrit. . . .

"Up to a few weeks ago, there was one man who alone of all mathematicians might have occupied the place which I hold without misgivings as to his fitness: I mean Henri Poincaré. It was at Rome just four years ago that the first dark shadow fell on us of that illness which has now terminated so fatally. You all remember the dismay which fell on us when the word passed from man to man, 'Poincaré is ill'. We had hoped that we might again have heard from his mouth some such luminous address as that which he gave at Rome; but it was not to be, and the loss of France in his death affects the whole world."*

In one of his essays Poincaré comes to the question of the object of science. He dignifies the old assertion, that the motive of our strivings is the desire for truth, by a few noble sentences: "When I speak here of truth, doubtless I have at first in mind scientific truth, but I would speak also of *la verité morale*, of which what we call justice is only one aspect. It would seem that I err in the use of words, that under one name I embrace two things with nothing in common, that *la verité scientifique* which is demonstrated cannot be linked with *la verité morale* which is felt. And yet I cannot separate them, and those who love the one love the other. To find the one, as to find the other, we must require ourselves to free our spirit of prejudice and passion, we must attain to absolute sincerity. These two kinds of truth, once discovered, bring us the same joy; each as soon as perceived glows with the same brilliancy and we must see it or close our eyes. Last of all, they both attract us and both flee from us; they are never fixed; when we think we have reached them, we see that we have further to go, and he who follows them is condemned never to know repose." The search for truth thus largely conceived, he insisted on as the goal of human activity, and the one end worthy of it. What he thus on one occasion so finely expressed in words, his whole life affirmed.

* As this paper is passing through the press, the word comes that death has claimed the distinguished scholar whose generous estimate of a fellow-worker is here quoted.